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Supersymmetric objects in the M-theory on a pp-wave

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Abstract: We obtain, in a systematic way, all the classical BPS equations which correspond to the quantum BPS states in the M-theory on a fully supersymmetric pp-wave. The superalgebra of the M-theory matrix model shows that the BPS states always preserve pairs of supersymmetry, implying the possible fractions of the unbroken supersymmetry as $\nu = 2/16, 4/16, 6/16, \ldots$ We study their classical counterparts, and find there are essentially one unique set of 2/16 BPS equations, three inequivalent types of 4/16 BPS equations, and three inequivalent types of 8/16 BPS equations only, in addition to the 16/16 static fuzzy sphere. We discuss various supersymmetric objects as solutions. In particular, when the fuzzy sphere rotates, the supersymmetry is further broken as $16/16 \to 8/16 \to 4/16$.

Keywords: M-Theory, D-branes, M(atrix) Theories.

Contents

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1. Introduction

At the present time, the most promising formalism for the description of the eleven dimensional M-theory prescribes the comactification on a light-like circle or on a small spatial circle boosted by a large amount, $x^- \sim x^- + 2\pi R$, as proposed by Banks, Fischler, Shenker and Susskind (BFSS) [1]–[4]. The sector of the theory with the discrete light cone momentum, $p_- = N/R$, is then exactly described in terms of D0-brane dynamics by the BFSS matrix model or the quantum mechanics obtained by dimensionally reducing the ten-dimensional U(N) super Yang-Mills theory. The BFSS matrix model is also in a good agreement with the matrix regularization of the supermembrane action in the light cone gauge [5, 6]. However, due to the flat directions in the potential the matrix model is of continuous spectrum and has proved very difficult to approach.

Recently [7], Berenstein, Maldacena and Nastase (BMN) showed that in the maximally supersymmetric pp-wave background of the eleven-dimensional supergravity [8, 9, 10],

$$
\begin{aligned}
ds^2 &= -2dx^+dx^- - \left[\left(\frac{\mu}{3}\right)^2 \left(x_1^2+x_2^2+x_3^2\right)+\left(\frac{\mu}{6}\right)^2 \left(x_4^2+\cdots+x_9^2\right)\right] dx^+dx^+ + \sum_{A=1}^{9} dx^A dx^A \,, \\
F_{123+} &= \mu \,,
\end{aligned}
\tag{1.1}
$$

the discrete light cone quantization (DLCQ) of the M-theory still works. With the characteristic mass parameter, μ, the resulting new matrix model corresponds to a mass deformation of the BFSS matrix model without breaking any supersymmetry. Soon after, the action was rederived as a description of the supermembrane on a pp-wave by Dasgupta, Sheikh-Jabbari and Van Raamsdonk [11].

Thanks to the mass parameter, the BMN matrix model captures many interesting novel properties. The supersymmetry transformations have explicit time dependence so that the supercharges do not commute with the hamiltonian. As a result, the bosons and fermions have different masses. The mass terms lift up the flat directions completely and the perturbative expansion is possible by powers of μ^{-1} [11, 12]. Classical vacua are given by fuzzy spheres sitting at the origin stretching over the $1,2,3$ directions.

In our previous work [13], we studied the superalgebra of this pp-wave matrix model. We identified the superalgebra as the special unitary Lie superalgebra, $\mathrm{su}(2|4\,;2,0)$ for $\mu > 0$ or $\mathrm{su}(2|4\,;2,4)$ for $\mu < 0$ of which the complexification corresponds to $\mathrm{A}(1|3)$. After analyzing its root structure, we discussed the typical and atypical representations deriving the "typicality" condition explicitly in terms of the energy and other four quantum numbers. In particular, we obtained the complete classification of the BPS multiplets which in general belong to a special class of the atypical unitary representations. They are classified as $4/16, 8/16, 12/16\,\mathrm{su}(2)$ singlet BPS multiplets and $8/16\,\mathrm{su}(4)$ singlet BPS multiplets, in addition to the $16/16$ vacua.

Generically, the BPS state is defined as a state in a supermultiplet which is annihilated by at least one Noether charge of the supersymmetry or one hermitean supercharge, while the BPS multiplet is defined as a unitary irreducible representation of which either the lowest weight or the highest weight is a BPS state. This definition naturally leads to a superspace with lower number of "odd" coordinates [14].

In [15], Dasgupta et al. investigated supermultiplets which contain at least one BPS state. These supermultiplets are not necessarily BPS multiplets. They concluded that there can appear $2/16, 4/16, 6/16, 8/16, 12/16, 16/16$ BPS states only in the supermultiplets.

Both of the analysis above were based on the pp-wave superalgebra which is free from the central charge. As the central charges in the matrix models appear as "the trace of the commutator", their absence is justified in the finite matrix models but not in the large-N limit.

In the present paper, we study the classical counterparts of the quantum BPS states. Namely we obtain all the classical BPS equations which correspond to the quantum BPS states in the BMN matrix model. Some simple analysis of the superalgebra which may now contain non-trivial central charges show that the BPS states always preserve pairs of supersymmetry, implying the possible fractions of the unbroken supersymmetry as $\nu = 2/16, 4/16, 6/16, \ldots$. Our main results are that there are essentially one unique set of $2/16$ BPS equations, three inequivalent types of $4/16$ BPS equations, and three inequivalent types of $8/16$ BPS equations only, in addition to the $16/16$ static fuzzy sphere. We discuss various supersymmetric objects as solutions. In particular, when the fuzzy sphere rotates on the transverse planes, the supersymmetry is further broken as $16/16 \to 8/16 \to 4/16$.

The key tool we employ here, following [16], is "the projection matrix" to the kernel space all the Killing spinors form. Once we are able to write down the projection matrix in terms of the anti-symmetric products of the gamma matrices, it is straightforward to obtain the corresponding BPS equations. In this way, the complete classification of the BPS equations in six and eight dimensions have been achieved in [16].

The organization of the present paper is as follows. In section 2, after setting up the gamma matrices and other conventions, we review the BMN matrix model. In particular, we write the action and the superalgebra in a so(3) $\times$ so(6) manifest fashion. All the possible central charges are included in this setup. Section 3 contains our main results. Firstly we discuss the general prescription to derive the classical BPS equations using the concept of the projection matrix. We then look at the quantum BPS states to identify the corresponding Killing spinors. Sequentially all the projection matrices relevant to the quantum BPS states are constructed and written in terms of the gamma matrix products. This enables us to obtain all the classical BPS equations corresponding to the quantum BPS states, the 2/16 BPS equations, the three types of 4/16 BPS equations, and the three types of 8/16 BPS equations. We discuss at least one class of solutions for each case. In section 4 we conclude with the summary. The appendix contains the BPS equations of the ten-dimensional super Yang-Mills theory as well as some useful formulae.

2. M-theory matrix model on a pp-wave

The action of the M-theory matrix model on a fully supersymmetric pp-wave background spells with a mass parameter, μ,

$$\mathcal{S} = \frac{l_p^6}{R^3}\int \mathrm{d}t\mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\,, \tag{2.1}$$

where with $i=1,2,3$, $a=4,5,\dots,9$, $A=1,2,\dots,9$,

$$\begin{aligned}
\mathcal{L}_0 &= \mathrm{tr}\left(\frac{1}{2}D_tX^AD_tX_A + \frac{1}{4}\left[X^A,X^B\right]^2 + i\frac{1}{2}\Psi^\dagger D_t\Psi - \frac{1}{2}\Psi^\dagger\Gamma^A[X_A,\Psi]\right),\\
\mathcal{L}_1 &= i\,\mathrm{tr}\left(-\frac{1}{3}\epsilon_{ijk}X^iX^jX^k + \frac{1}{8}\Psi^\dagger\Gamma^{123}\Psi\right),\\
\mathcal{L}_2 &= -\frac{1}{2}\,\mathrm{tr}\left(\left(\frac{1}{3}\right)^2X^iX_i + \left(\frac{1}{6}\right)^2X^aX_a\right).
\end{aligned} \tag{2.2}$$

We make a few remarks especially compared to the original one given by BMN [7]. Here the euclidean nine-dimensional gamma matrices, $\Gamma^A = (\Gamma^A)^\dagger$, are generic ones. Namely we do not adopt the usual real and symmetric Majorana representation. Accordingly there exits a non-trivial 16×16 charge conjugation matrix, C,

$$(\Gamma^A)^T = (\Gamma^A)^* = C^{-1}\Gamma^AC\,, \qquad C = C^T = (C^\dagger)^{-1}\,. \tag{2.3}$$

The spinor, Ψ, satisfies the Majorana condition leaving eight independent complex components

$$\Psi = C\Psi^*\,. \tag{2.4}$$

The covariant derivatives are in our convention, $D_t\mathcal{O} = \frac{d}{dt}\mathcal{O} - i[A_0, \mathcal{O}]$ so that X and A_0 are of the mass dimension one, while Ψ has the mass dimension $3/2$. The overall constant, l_p^6/R^3, is set to be one henceforth.

The supersymmetry transformations are

$$\begin{aligned}
\delta A_0 &= i\Psi^\dagger \mathcal{E}(t)\,, \qquad \delta X^A = i\Psi^\dagger \Gamma^A \mathcal{E}(t)\,, \\
\delta\Psi &= \left(D_t X^A \Gamma_A - i\frac{1}{2}\left[X^A, X^B\right]\Gamma_{AB} + \mu\left(-\frac{1}{3}X^i\Gamma_i + \frac{1}{6}X^a\Gamma_a\right)\Gamma^{123}\right)\mathcal{E}(t)\,,
\end{aligned} \tag{2.5}$$

where

$$\mathcal{E}(t) = e^{\frac{\mu}{12}\Gamma^{123}t}\mathcal{E}\,, \qquad \mathcal{E} = C\mathcal{E}^*\,. \tag{2.6}$$

In addition there is kinetic supersymmetry,

$$\delta A_0 = \delta X^A = 0\,, \qquad \delta\Psi = e^{-\frac{\mu}{4}\Gamma^{123}t}\mathcal{E}'\,, \qquad \mathcal{E}' = C\mathcal{E}'^*\,. \tag{2.7}$$

2.1 Manifestation of the $\mathrm{so}(3)\times\mathrm{so}(6)$ structure

To make the $\mathrm{so}(3)\times\mathrm{so}(6) \equiv \mathrm{su}(2)\times\mathrm{su}(4)$ structure of the M-theory on a pp-wave manifest, we write the nine-dimensional gamma matrices in terms of the three and six-dimensional ones, σ^i, γ^a,

$$\begin{aligned}
\Gamma^i &= \sigma^i \otimes \gamma^{(7)}\,, \qquad \text{for } i = 1,2,3\,, \\
\Gamma^a &= 1 \otimes \gamma^a\,, \qquad \text{for } a = 4,5,6,7,8,9\,.
\end{aligned} \tag{2.8}$$

With the choice,

$$\gamma^{(7)} = i\gamma^4\gamma^5\ldots\gamma^9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.9}$$

the six-dimensional gamma matrices are in the block diagonal form

$$\gamma^a = \begin{pmatrix} 0 & \rho^a \\ \bar\rho^a & 0 \end{pmatrix}, \qquad \rho^a\bar\rho^b + \rho^b\bar\rho^a = 2\delta^{ab}\,. \tag{2.10}$$

The fact, $\bar\rho^a = (\rho^a)^\dagger$, ensures γ^a to be hermitean. Furthermore, it is possible to set the 4×4 matrices, ρ^a, to be anti-symmetric [17]

$$(\rho^a)_{\dot\alpha\dot\beta} = -(\rho^a)_{\dot\beta\dot\alpha}\,, \qquad (\bar\rho^a)^{\dot\alpha\dot\beta} = -(\bar\rho^a)^{\dot\beta\dot\alpha}\,. \tag{2.11}$$

Namely six of ρ_a form a basis of the 4×4 anti-symmetric matrices. It follows that fifteen of $\rho_{ab} = \rho_{[a}\bar\rho_{b]}$ and ten of $\rho_{abc} = \rho_{[a}\bar\rho_b\rho_{c]}$ form basis of the traceless and symmetric matrices, respectively. The latter is subject to the self-duality,

$$\rho_{abc} = \frac{i}{6}\epsilon_{abcdef}\rho^{def}\,. \tag{2.12}$$

Henceforth $\alpha, \beta = 1, 2$ are the su(2) indices, and $\dot\alpha, \dot\beta$ denote the su(4) indices, $1, 2, 3, 4$.

The nine-dimensional charge conjugation matrix, C, is now explicitly

$$C = \epsilon \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad (\sigma^i)^T = -\epsilon^{-1}\sigma^i\epsilon\,, \tag{2.13}$$

so that Majorana spinors contain eight independent complex components,

$$\Psi = \begin{pmatrix} \psi_{\alpha\dot\alpha} \\ \tilde\psi_\alpha{}^{\dot\alpha} \end{pmatrix}, \qquad \tilde\psi_\alpha{}^{\dot\alpha} = \epsilon_{\alpha\beta}(\psi^*)^{\beta\dot\alpha},$$
$$\mathcal{E}(t) = \begin{pmatrix} e^{i\frac{\mu}{12}t}\varepsilon_{\alpha\dot\alpha} \\ e^{-i\frac{\mu}{12}t}\tilde\varepsilon_\alpha{}^{\dot\alpha} \end{pmatrix}, \qquad \tilde\varepsilon_\alpha{}^{\dot\alpha} = \epsilon_{\alpha\beta}(\varepsilon^*)^{\beta\dot\alpha}. \tag{2.14}$$

We rewrite the lagrangian, $\mathcal{L}_0, \mathcal{L}_1$, in a more so(3) $\times$ so(6) manifest fashion,

$$\begin{aligned}\mathcal{L}_0 &= \mathrm{tr}\left(\frac{1}{2}D_tX^AD_tX_A + \frac{1}{4}\left[X^A,X^B\right]^2 + i\bar\psi D_t\psi - \right.\\ &\qquad \left. - \bar\psi\sigma^i[X_i,\psi] - \frac{1}{2}\bar\psi\rho^a\left[X_a,\tilde\psi\right] - \frac{1}{2}\bar{\tilde\psi}\bar\rho^a[X_a,\psi]\right),\\ \mathcal{L}_1 &= -\,\mathrm{tr}\left(i\frac{1}{3}\epsilon_{ijk}X^iX^jX^k + \frac{1}{4}\bar\psi\psi\right).\end{aligned} \tag{2.15}$$

The supersymmetry transformation (2.5) becomes

$$\begin{aligned}\delta A_0 &= i\left(\bar\psi\varepsilon(t) - \bar\varepsilon(t)\psi\right), \qquad \delta X^i = i\left(\bar\psi\sigma^i\varepsilon(t) - \bar\varepsilon(t)\sigma^i\psi\right),\\ \delta X^a &= i\left(\bar{\tilde\psi}\bar\rho^a\varepsilon(t) - \bar\varepsilon(t)\rho^a\tilde\psi\right),\\ \delta\psi &= \left(D_tX^i\sigma_i - i\frac{1}{2}[X^i,X^j]\sigma_{ij} - i\frac{1}{2}\left[X^a,X^b\right]\rho_{ab} - i\frac{\mu}{3}X^i\sigma_i\right)\varepsilon(t) + \\ &\quad + \left(D_tX^a\rho_a - i[X^i,X^a]\sigma_i\rho_a - i\frac{\mu}{6}X^a\rho_a\right)\tilde\varepsilon(t),\end{aligned} \tag{2.16}$$

where $\varepsilon(t) = e^{i\frac{\mu}{12}t}\varepsilon$ and $\bar\psi = \psi^\dagger$, etc.

The Gauss constraint reads with $P^A = D_tX^A$,

$$i[X^A,P_A] + \{\bar\psi^{\alpha\dot\alpha},\psi_{\alpha\dot\alpha}\} = 0, \tag{2.17}$$

while the equations of motion are given in the appendix (A.6).

The hamiltonian and the so(3) $\times$ so(6) angular momenta are explicitly

$$\begin{aligned}H &= \mathrm{tr}\left(\frac{1}{2}P^AP_A - \frac{1}{4}\left[X^A,X^B\right]^2 + \bar\psi\sigma^i[X_i,\psi] + \frac{1}{2}\bar\psi\rho^a\left[X_a,\tilde\psi\right] + \frac{1}{2}\bar{\tilde\psi}\bar\rho^a[X_a,\psi] + \right.\\ &\quad \left. + i\frac{\mu}{3}\epsilon_{ijk}X^iX^jX^k + \frac{\mu}{4}\bar\psi\psi + \frac{1}{2}\left(\frac{\mu}{3}\right)^2X^iX_i + \frac{1}{2}\left(\frac{\mu}{6}\right)^2X^aX_a\right),\end{aligned} \tag{2.18}$$

$$M^{ij} = \mathrm{tr}\left(X^iP^j - P^iX^j - \frac{i}{2}\bar\psi\sigma^{ij}\psi\right), \qquad M^{ab} = \mathrm{tr}\left(X^aP^b - P^aX^b - \frac{i}{2}\bar\psi\rho^{ab}\psi\right). \tag{2.19}$$

2.2 Supersymmetry algebra

The Noether charge of the supersymmetry is, from (A.1), of the form

$$i\ \mathrm{tr}\left(\Psi^\dagger\delta\Psi\right) = \bar\varepsilon^{\alpha\dot\alpha}Q_{\alpha\dot\alpha} + \bar Q^{\alpha\dot\alpha}\varepsilon_{\alpha\dot\alpha}, \tag{2.20}$$

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where the eight component supercharges are with $\bar{Q}^{\alpha\dot{\alpha}} = (Q_{\alpha\dot{\alpha}})^{\dagger}$

$$Q = -ie^{-i\frac{\mu}{12}t}\,\mathrm{tr}\left[\left(\left(P^i + i\frac{\mu}{3}X^i\right)\sigma_i + i\frac{1}{2}[X^i, X^j]\sigma_{ij} + i\frac{1}{2}\left[X^a, X^b\right]\rho_{ab}\right)\psi + \right.$$
$$\left. + \left(\left(P^a - i\frac{\mu}{6}X^a\right)\rho_a + i[X^i, X^a]\sigma_i\rho_a\right)\tilde{\psi}\right]. \qquad (2.21)$$

After the standard quantization,

$$\left[X^{Al}{}_m, P^{Bn}{}_r\right] = i\delta^{AB}\delta^l{}_r\delta_m{}^n\,, \qquad \left\{\psi_{\alpha\dot{\alpha}}{}^l{}_m, \bar{\psi}^{\beta\dot{\beta}n}{}_r\right\} = \delta_\alpha{}^\beta\delta_{\dot{\alpha}}{}^{\dot{\beta}}\delta^l{}_r\delta_m{}^n\,, \qquad (2.22)$$

using (A.11), one can identify the supersymmetry algebra of the M-theory on a fully supersymmetric pp-wave as follows up to the Gauss constraint (cf. [18, 19, 20])

$$[H, Q_{\alpha\dot{\alpha}}] = \frac{\mu}{12}Q_{\alpha\dot{\alpha}}\,, \qquad [H, \bar{Q}^{\alpha\dot{\alpha}}] = -\frac{\mu}{12}\bar{Q}^{\alpha\dot{\alpha}}\,, \qquad (2.23)$$

$$[M_{ij}, Q_{\alpha\dot{\alpha}}] = i\frac{1}{2}(\sigma_{ij})_\alpha{}^\beta Q_{\beta\dot{\alpha}}\,, \qquad [M_{ab}, Q_{\alpha\dot{\alpha}}] = i\frac{1}{2}(\rho_{ab})_{\dot{\alpha}}{}^{\dot{\beta}}Q_{\alpha\dot{\beta}}\,,$$
$$[M_{ij}, \bar{Q}^{\alpha\dot{\alpha}}] = -i\frac{1}{2}\bar{Q}^{\beta\dot{\alpha}}(\sigma_{ij})_\beta{}^\alpha\,, \qquad [M_{ab}, \bar{Q}^{\alpha\dot{\alpha}}] = -i\frac{1}{2}\bar{Q}^{\alpha\dot{\beta}}(\rho_{ab})_{\dot{\beta}}{}^{\dot{\alpha}}\,, \qquad (2.24)$$

$$[M_i, M_j] = i\epsilon_{ijk}M^k\,, \qquad M_i = \frac{1}{2}\epsilon_{ijk}M^{jk}\,,$$
$$[M_{ab}, M_{cd}] = i(\delta_{ac}M_{bd} - \delta_{ad}M_{bc} - \delta_{bc}M_{ad} + \delta_{bd}M_{ac})\,, \qquad (2.25)$$

$$\left\{Q_{\alpha\dot{\alpha}}, \bar{Q}^{\beta\dot{\beta}}\right\} = 2\delta_\alpha{}^\beta\delta_{\dot{\alpha}}{}^{\dot{\beta}}H + i\frac{\mu}{3}(\sigma^{ij})_\alpha{}^\beta\delta_{\dot{\alpha}}{}^{\dot{\beta}}M_{ij} - i\frac{\mu}{6}\delta_\alpha{}^\beta(\rho^{ab})_{\dot{\alpha}}{}^{\dot{\beta}}M_{ab} + \frac{1}{4}(\sigma^{ij})_\alpha{}^\beta(\rho^{ab})_{\dot{\alpha}}{}^{\dot{\beta}}R_{ijab}\,,$$
$$\left\{Q_{\alpha\dot{\alpha}}, Q_{\beta\dot{\beta}}\right\} = \epsilon_{\alpha\beta}(\rho^a)_{\dot{\alpha}\dot{\beta}}Z_a + \frac{1}{6}(\sigma^i\epsilon)_{\alpha\beta}(\rho^{abc})_{\dot{\alpha}\dot{\beta}}Z_{iabc}\,, \qquad (2.26)$$

where $R_{ij\,ab}, Z_a, Z_{iabc}$ are central charges given by the “boundary terms” or the trace of the commutator. Surely they vanish for the finite matrix models. $R_{ij\,ab}$ and Z_{iabc} satisfy the reality and the anti-self-duality conditions, respectively,

$$R_{ij\,ab} = -R_{ji\,ab} = -R_{ij\,ba} = (R_{ij\,ab})^{\dagger}\,, \qquad Z_{iabc} = -\frac{i}{6}\epsilon_{abcdef}Z_i{}^{def}\,. \qquad (2.27)$$

Note that the numbers of degrees of the left and right sides in (2.26) match as

$$8\times 8 = 1 + 3 + 15 + 3\times 15\,,$$
$$36 = 6 + 3\times 10\,. \qquad (2.28)$$

Basically they are the decompositions of 8×8 hermitean and symmetric matrices in terms of σ^i and $\rho^a, \bar{\rho}^b$.

As shown in our previous work [13], in the absence of the central charges, the above superalgebra of the M-theory on a fully supersymmetric pp-wave is identified as the special unitary Lie superalgebra, $\mathrm{su}(2|4\,;2,0)$ for $\mu > 0$ or $\mathrm{su}(2|4\,;2,4)$ for $\mu < 0$, the complexification of which corresponds to $\mathrm{A}(1|3)$.

A natural choice of the Cartan subalgebra, $\mathrm{u}(1)\oplus\mathrm{su}(2)\oplus\mathrm{su}(4)$, is

$$\{H, M_{12}, M_{45}, M_{67}, M_{89}\}\,. \qquad (2.29)$$

Any state in a supermultiplet or an irreducible representation of the superalgebra is specified by the quantum numbers of the Cartan subalgebra, while all the states in a supermultiplet carry the same central charges. References [13] contains the complete classification of the irreducible representations of $\mathrm{A}(1|3)$.

3. Supersymmetric objects

Classically a bosonic configuration is supersymmetric or BPS if there exits a non-zero constant Killing spinor, $\mathcal{E}$, such that the infinitesimal supersymmetric transformation of the gaugino vanishes,

$$\delta\Psi = \left(D_t X^A\Gamma_A - i\frac{1}{2}[X^A, X^B]\Gamma_{AB} + \mu\left(-\frac{1}{3}X^i\Gamma_i + \frac{1}{6}X^a\Gamma_a\right)\Gamma^{123}\right)e^{\frac{\mu}{12}\Gamma^{123}t}\mathcal{E} = 0\,. \tag{3.1}$$

In general, a BPS configuration can have more than one Killing spinors. The key tool we employ here, following [16], is the projection matrix to the kernel space, V, all the Killing spinors form. With an orthonormal basis for the kernel, $V = \{\mathcal{E}_n | 1 \leq n \leq \mathcal{N}\}$, $\mathcal{N} = \dim V \leq 16$, the projection operator is formally

$$\Omega = \sum_{n=1}^{N} \mathcal{E}_n \mathcal{E}_n^\dagger\,, \tag{3.2}$$

and satisfies[1]

$$\Omega^\dagger = \Omega\,, \qquad C\Omega^* C^{-1} = \Omega\,, \tag{3.3}$$

$$\Omega^2 = \Omega\,, \tag{3.4}$$

$$\left(D_t X^A\Gamma_A - i\frac{1}{2}\left[X^A, X^B\right]\Gamma_{AB} + \mu\left(-\frac{1}{3}X^i\Gamma_i + \frac{1}{6}X^a\Gamma_a\right)\Gamma^{123}\right)e^{\frac{\mu}{12}\Gamma^{123}t}\Omega = 0\,. \tag{3.5}$$

It is worth to note that Ω is basis independent or unique for a given BPS configuration.

As the anti-symmetric products of the gamma matrices form a basis of the 16×16 matrices, one can expand Ω in terms of them. Up to the relation, $\Gamma^{12\cdots 9} = 1$, eq. (3.3) restricts the projection matrix to be of the form

$$\Omega = \nu\left(1 + r_A\Gamma^A + \frac{1}{4!}r_{ABCD}\Gamma^{ABCD}\right), \tag{3.6}$$

where r_A and r_{ABCD} are real one and four form coefficients, while ν denotes the fraction of the unbroken supersymmetry. As the eigenvalues of Ω are either 0 or 1 and the non-trivial products of the gamma matrices are traceless,

$$16 \times \nu = \mathrm{tr}\,\Omega = N\,. \tag{3.7}$$

The only equation left to solve is (3.4) in order to get the final form of the projection operator. Once it is done, the BPS equations follow straightforwardly from expanding (3.5) by the anti-symmetric products of the gamma matrices and requiring each coefficient to vanish. However, we do not know the most general solution of (3.4). Unlike the cases

[1] For simplicity we do not turn on the kinetic supersymmetry transformations which may cancel the dynamic supersymmetry transformations in the large-N limit.

in four, six and eight dimensions [16], the present isometry group, SO(3) $\times$ SO(6), is not big enough to reduce the number of free parameters significantly to give the essentially unique solution. It appears there are infinitely many classical BPS equations which are not equivalent, even up to the isometry group.

However, this is a genuinely classical problem. Once we consider the quantum aspects or the BPS states, the classical complexity gets cleaned up and one can identify all the projection matrices or all the classical BPS equations relevant to the quantum BPS states.

3.1 Quantum aspects

The *BPS state*, $|\Psi_{\rm BPS}\rangle$, is defined as a state in a supermultiplet which is annihilated by at least one Noether charge of the supersymmetry or one hermitean supercharge,

$$\left(\bar{\varepsilon}^{\alpha\dot{\alpha}}Q_{\alpha\dot{\alpha}}+\bar{Q}^{\alpha\dot{\alpha}}\varepsilon_{\alpha\dot{\alpha}}\right)|\Psi_{\rm BPS}\rangle=0\,. \tag{3.8}$$

The corresponding sixteen component Killing spinor is

$$\mathcal{E}=\begin{pmatrix}\varepsilon\\ \epsilon\varepsilon^{*}\end{pmatrix}. \tag{3.9}$$

One crucial step we take here is to diagonalize $\Gamma^{12},\Gamma^{45},\Gamma^{67},\Gamma^{89}$ which are for the Cartan subalgebra we chose (2.29). This is done by using the U(4) symmetry, $\rho^{a}\rightarrow U\rho^{a}U^{T}, UU^{\dagger}=1$, which preserves the anti-symmetric property of ρ^{a}. The appendix contains explicitly an example of such gamma matrices (A.10).

Now, as seen in (2.24), each of the sixteen supercharges carries definite quantum numbers of the Cartan subalgebra. In fact,[2] any four generators including M_{12} in the Cartan subalgebra can uniquely specify all the sixteen supercharges by their quantum numbers, essentially as $16=2^{4}$. Accordingly we have for all the $(\alpha,\dot{\alpha})$ pairs satisfying $\varepsilon_{\alpha\dot{\alpha}}\neq0$,

$$Q_{\alpha\dot{\alpha}}|\Psi_{\rm BPS}\rangle=0\,,\qquad\bar{Q}^{\alpha\dot{\alpha}}|\Psi_{\rm BPS}\rangle=0\,, \tag{3.10}$$

or equivalently[3]

$$(Q_{\alpha\dot{\alpha}}+\bar{Q}^{\alpha\dot{\alpha}})|\Psi_{\rm BPS}\rangle=0\,,\qquad i(Q_{\alpha\dot{\alpha}}-\bar{Q}^{\alpha\dot{\alpha}})|\Psi_{\rm BPS}\rangle=0\,. \tag{3.11}$$

Namely, in the M-theory matrix model on a pp-wave, the BPS state always preserves pairs of supersymmetry, implying the possible fractions of the unbroken supersymmetry as $\nu=2/16,4/16,6/16,\ldots,16/16$.

If we introduce a basis for eight component spinors, $\{\zeta^{\alpha\dot{\alpha}}\}$, such that their $(\beta,\dot{\beta})$ components are

$$(\zeta^{\alpha\dot{\alpha}})_{\beta\dot{\beta}}=\delta^{\alpha}{}_{\beta}\delta^{\dot{\alpha}}{}_{\dot{\beta}}\,, \tag{3.12}$$

we can write the pair of Killing spinors for (3.11),

$$\mathcal{E}^{\alpha\dot{\alpha}}_{+}=\frac{1}{\sqrt{2}}\begin{pmatrix}\zeta^{\alpha\dot{\alpha}}\\ \epsilon(\zeta^{\alpha\dot{\alpha}})^{*}\end{pmatrix},\qquad\mathcal{E}^{\alpha\dot{\alpha}}_{-}=\frac{1}{\sqrt{2}}\begin{pmatrix}i\zeta^{\alpha\dot{\alpha}}\\ -i\epsilon(\zeta^{\alpha\dot{\alpha}})^{*}\end{pmatrix}=\Gamma^{123}\mathcal{E}^{\alpha\dot{\alpha}}_{+}. \tag{3.13}$$

[2] This justifies the generality of the BPS equations of the ordinary ten-dimensional super Yang-Mills theory we obtain in (A.12).

[3] One can also easily check from (2.23) that one annihilation in (3.11) implies the other.

The corresponding $\nu = 2/16$ projection matrix is then

$$\Omega_{\alpha\dot\alpha} = \mathcal{E}_{+}^{\alpha\dot\alpha}(\mathcal{E}_{+}^{\alpha\dot\alpha})^{\dagger} + \mathcal{E}_{-}^{\alpha\dot\alpha}(\mathcal{E}_{-}^{\alpha\dot\alpha})^{\dagger} = \begin{pmatrix} \zeta^{\alpha\dot\alpha}(\zeta^{\alpha\dot\alpha})^{T} & 0 \\ 0 & \epsilon\zeta^{\alpha\dot\alpha}(\zeta^{\alpha\dot\alpha})^{T}\epsilon^{-1} \end{pmatrix}. \tag{3.14}$$

From (A.10) it is straightforward to expand this projection matrix in terms of the gamma matrices

$$\begin{aligned} \Omega_{\alpha\dot\alpha} = \frac{1}{8}\Big(&1 + \lambda_0\Gamma^{3} - \lambda_1\Gamma^{6789} - \lambda_2\Gamma^{8945} - \lambda_1\lambda_2\Gamma^{4567} - \\ &- \lambda_0\lambda_1\Gamma^{1245} - \lambda_0\lambda_2\Gamma^{1267} - \lambda_0\lambda_1\lambda_2\Gamma^{1289}\Big). \end{aligned} \tag{3.15}$$

Here $\lambda_0, \lambda_1, \lambda_2$ are three independent signs,

$$\lambda_0^2 = \lambda_1^2 = \lambda_2^2 = 1\,, \tag{3.16}$$

which are related to $(\alpha, \dot\alpha)$ or the unbroken supersymmetry, $Q_{\alpha\dot\alpha} + \bar{Q}^{\alpha\dot\alpha}$, $i(Q_{\alpha\dot\alpha} - \bar{Q}^{\alpha\dot\alpha})$, as

λ_0	λ_1	λ_2	$(\alpha, \dot\alpha)$
+	+	+	$(1,1)$
+	+	−	$(1,2)$
+	−	+	$(1,3)$
+	−	−	$(1,4)$
−	+	+	$(2,1)$
−	+	−	$(2,2)$
−	−	+	$(2,3)$
−	−	−	$(2,4)$

This relation enables us to define $\Omega_\lambda \equiv \Omega_{\alpha\dot\alpha}$, $\lambda = (\lambda_0, \lambda_1, \lambda_2)$. It is worth to note that Ω_λ's are orthogonal to each other

$$\Omega_\lambda\Omega_{\lambda'} = \delta_{\lambda\lambda'}\Omega_\lambda\,, \tag{3.17}$$

and also complete

$$\sum_\lambda \Omega_\lambda = 1_{16\times 16}\,. \tag{3.18}$$

Other generic projection matrices of the fractions, $\nu = \mathcal{N}/16$, $\mathcal{N} = 4, 6, \ldots, 16$, are then constructed by summing $\mathcal{N}/2$ different $2/16$ projection operators above.

3.2 2/16 BPS configurations

Substituting (3.15) into (3.5) and expanding it by the anti-symmetric products of the gamma matrices, it is straightforward to obtain the 2/16 BPS equations,

$$\begin{aligned} &D_tX^1 + i\lambda_0\left[X^3, X^1\right] + \lambda_0\frac{\mu}{3}X^2 = 0\,, \qquad D_tX^2 + i\lambda_0\left[X^3, X^2\right] - \lambda_0\frac{\mu}{3}X^1 = 0\,, \\ &D_tX^3 = 0\,, \\ &D_tX^4 + i\lambda_0\left[X^3, X^4\right] - \lambda_1\frac{\mu}{6}X^5 = 0\,, \qquad D_tX^5 + i\lambda_0\left[X^3, X^5\right] + \lambda_1\frac{\mu}{6}X^4 = 0\,, \end{aligned}$$

$$\begin{aligned}
D_tX^6 + i\lambda_0\left[X^3, X^6\right] - \lambda_2\frac{\mu}{6}X^7 &= 0\,, & D_tX^7 + i\lambda_0\left[X^3, X^7\right] + \lambda_2\frac{\mu}{6}X^6 &= 0\,,\\
D_tX^8 + i\lambda_0\left[X^3, X^8\right] - \lambda_1\lambda_2\frac{\mu}{6}X^9 &= 0\,, & D_tX^9 + i\lambda_0\left[X^3, X^9\right] + \lambda_1\lambda_2\frac{\mu}{6}X^8 &= 0\,,\\
\left[X^1, X^4\right] - \lambda_0\lambda_1\left[X^2, X^5\right] &= 0\,, & \left[X^1, X^5\right] + \lambda_0\lambda_1\left[X^2, X^4\right] &= 0\,,\\
\left[X^1, X^6\right] - \lambda_0\lambda_2\left[X^2, X^7\right] &= 0\,, & \left[X^1, X^7\right] + \lambda_0\lambda_2\left[X^2, X^6\right] &= 0\,,\\
\left[X^1, X^8\right] - \lambda_0\lambda_1\lambda_2\left[X^2, X^9\right] &= 0\,, & \left[X^1, X^9\right] + \lambda_0\lambda_1\lambda_2\left[X^2, X^8\right] &= 0\,,\\
\left[X^4, X^6\right] - \lambda_1\lambda_2\left[X^5, X^7\right] &= 0\,, & \left[X^4, X^7\right] + \lambda_1\lambda_2\left[X^5, X^6\right] &= 0\,,\\
\left[X^4, X^8\right] - \lambda_2\left[X^5, X^9\right] &= 0\,, & \left[X^4, X^9\right] + \lambda_2\left[X^5, X^8\right] &= 0\,,\\
\left[X^6, X^8\right] - \lambda_1\left[X^7, X^9\right] &= 0\,, & \left[X^6, X^9\right] + \lambda_1\left[X^7, X^8\right] &= 0\,,
\end{aligned}$$

$$\lambda_0\left[X^1, X^2\right] + \lambda_1\left[X^4, X^5\right] + \lambda_2\left[X^6, X^7\right] + \lambda_1\lambda_2\left[X^8, X^9\right] - i\lambda_0\frac{\mu}{3}X^3 = 0\,. \tag{3.19}$$

In addition there exists the Gauss constraint

$$\sum_A\left[D_tX^A, X_A\right] = 0\,. \tag{3.20}$$

Surely any solution of the BPS equations subject to the Gauss constraint satisfies the full equations of motion (A.6), as discussed in the appendix.

All the different choices for $\lambda = (\lambda_0, \lambda_1, \lambda_2)$ are $\mathrm{SO}(3)\times\mathrm{SO}(6)$ equivalent. In particular, for $\lambda = (+,+,+)$, if we complexify the coordinates as

$$Z_0 = X_1 + iX_2\,, \qquad Z_1 = X_4 + iX_5\,, \qquad Z_2 = X_6 + iX_7\,, \qquad Z_3 = X_8 + iX_9\,, \tag{3.21}$$

and set $\bar{Z}_\mu = (Z_\mu)^\dagger, \mu = 0, 1, 2, 3$, the 2/16 BPS equations get simplified as

$$\begin{aligned}
D_tZ_0 + i[X^3, Z_0] - i\frac{\mu}{3}Z_0 &= 0\,, \qquad D_tZ_1 + i[X^3, Z_1] + i\frac{\mu}{6}Z_1 = 0\,,\\
D_tZ_2 + i[X^3, Z_2] + i\frac{\mu}{6}Z_2 &= 0\,, \qquad D_tZ_3 + i[X^3, Z_3] + i\frac{\mu}{6}Z_3 = 0\,,\\
D_tX^3 &= 0\,, \qquad [Z_\mu, Z_\nu] = 0\,,\\
\left[Z_0, \bar{Z}_0\right] + \left[Z_1, \bar{Z}_1\right] + \left[Z_2, \bar{Z}_2\right] &+ \left[Z_3, \bar{Z}_3\right] - \frac{2}{3}\mu X^3 = 0\,.
\end{aligned} \tag{3.22}$$

The Gauss constraint reads

$$\sum_{\mu=0}^{3}\left[D_tZ_\mu, \bar{Z}_\mu\right] + \left[D_t\bar{Z}_\mu, Z_\mu\right] = 0\,. \tag{3.23}$$

The energy is saturated, from (2.26) and (A.9), by the angular momenta and the central charges,

$$H = \frac{\mu}{3}M_{12} - \frac{\mu}{6}\left(M_{45} + M_{67} + M_{89}\right) + \frac{1}{2}\left(R_{1245} + R_{1267} + R_{1289}\right). \tag{3.24}$$

Solutions of the finite size surely do not carry any central charge so that they describe rotating supersymmetric objects. On the other hand, from the Hodge duality, $R_{iab} \equiv \frac{1}{2}\epsilon_{ijk}R_{jkab}$, the non-trivial static solutions correspond either to the longitudinal large M5-

branes stretching in the $1,2$ and at least two of $4,5,6,7,8,9$ directions or to the large membranes stretching in the third and at least two of $4,5,6,7,8,9$ directions.

Though we do not know the most general solutions, it is easy to find solutions for the D0-branes rotating in the whole space except the third direction,

$$\begin{aligned} Z_0(t) &= e^{i\frac{\mu}{3}t}\mathcal{Z}_0\,, \qquad Z_1(t) = e^{-i\frac{\mu}{6}t}\mathcal{Z}_1\,, \\ Z_2(t) &= e^{-i\frac{\mu}{6}t}\mathcal{Z}_2\,, \qquad Z_3(t) = e^{-i\frac{\mu}{6}t}\mathcal{Z}_3\,, \\ A_0 &= X^3 = 0\,, \qquad \mathcal{Z}_\mu : \text{diagonal matrices}\,. \end{aligned} \tag{3.25}$$

3.3 4/16 BPS configurations — type I

The $\nu = 4/16$ projection matrices are sum of any two different 2/16 projection operators. It is easy to see that there are three inequivalent ways of summing. The first type we consider corresponds to the choice, $\lambda = (+++)$ and $(++-)$. The relevant 4/16 BPS equations are

$$\begin{aligned} D_tZ_0 + i\left[X^3, Z_0\right] - i\frac{\mu}{3}Z_0 &= 0\,, \qquad D_tZ_1 + i\left[X^3, Z_1\right] + i\frac{\mu}{6}Z_1 = 0\,, \\ D_tX^3 &= 0\,, \qquad [Z_0, Z_1] = 0\,, \\ X^6 = X^7 = X^8 = X^9 &= 0\,, \qquad \left[Z_0, \bar{Z}_0\right] + \left[Z_1, \bar{Z}_1\right] - \frac{2}{3}\mu X^3 = 0\,. \end{aligned} \tag{3.26}$$

The Gauss constraint reads

$$\left[D_tZ_0, \bar{Z}_0\right] + \left[D_t\bar{Z}_0, Z_0\right] + \left[D_tZ_1, \bar{Z}_1\right] + \left[D_t\bar{Z}_1, Z_1\right] = 0\,. \tag{3.27}$$

The energy is saturated by the angular momenta as well as the central charges,

$$H = \frac{\mu}{3}M_{12} - \frac{\mu}{6}M_{45} + \frac{1}{2}R_{1245}\,, \tag{3.28}$$

and $M_{67} = M_{89} = R_{1267} = R_{1289} = 0$.

Again, without knowing the most general solutions, we write the D0-brane solutions rotating on the $(1,2)$ and $(4,5)$ planes,

$$\begin{aligned} Z_0(t) &= e^{i\frac{\mu}{3}t}\mathcal{Z}_0\,, \qquad Z_1(t) = e^{-i\frac{\mu}{6}t}\mathcal{Z}_1\,, \\ A_0 &= X^3 = X^6 = X^7 = X^8 = X^9 = 0\,, \qquad \mathcal{Z}_0,\ \mathcal{Z}_1 : \text{diagonal matrices}\,. \end{aligned} \tag{3.29}$$

3.4 4/16 BPS configurations — type II: su(2) singlet, rotating fuzzy sphere

The choice, $\lambda = (+++)$ and $(-++)$, corresponds quantum mechanically to the 4/16 su(2) singlet BPS multiplet [13]. The energy is saturated by the angular momenta only,

$$H = -\frac{\mu}{6}(M_{45} + M_{67} + M_{89})\,, \tag{3.30}$$

since $M_{12} = R_{1245} = R_{1267} = R_{1289} = 0$. The relevant 4/16 BPS equations are

$$\begin{aligned} [X_i, X_j] - i\frac{\mu}{3}\epsilon_{ijk}X^k &= 0\,, \qquad D_tX^i = 0\,, \qquad [X^i, X^a] = 0\,, \\ D_tZ_1 + i\frac{\mu}{6}Z_1 &= 0\,, \qquad D_tZ_2 + i\frac{\mu}{6}Z_2 = 0\,, \qquad D_tZ_3 + i\frac{\mu}{6}Z_3 = 0\,, \\ [Z_1, Z_2] &= 0\,, \qquad [Z_2, Z_3] = 0\,, \qquad [Z_3, Z_1] = 0\,, \\ \left[Z_1, \bar{Z}_1\right] + \left[Z_2, \bar{Z}_2\right] + \left[Z_3, \bar{Z}_3\right] &= 0 : \text{Gauss constraint}\,. \end{aligned} \tag{3.31}$$

Note that the BPS equations themselves contain the Gauss constraint in this case.

For the finite matrices, the BPS equations imply that Z_1, Z_2, Z_3 can be simultaneously triangulized. Then the Gauss constraint tells us that they are actually diagonal. Therefore, the above BPS equations describe the fuzzy sphere or the giant graviton rotating on the $(4,5),(6,7)$ and $(8,9)$ planes,

$$\begin{aligned} X_i &= \frac{\mu}{3}J_i\,, \qquad [J_i,J_j]=i\epsilon_{ijk}J_k\,, \qquad A_0=0\,,\\ Z_1(t) &= e^{-i\frac{\mu}{6}t}z_1 1\,, \qquad Z_2(t)=e^{-i\frac{\mu}{6}t}z_2 1\,, \qquad Z_3(t)=e^{-i\frac{\mu}{6}t}z_3 1\,, \end{aligned} \tag{3.32}$$

where z_1, z_2, z_3 are arbitrary complex numbers indicating the position of the fuzzy sphere at $t=0$. This gives the most general irreducible finite matrix solutions.

On the other hand, in the large-N limit, by setting $X^i = Z_3 = A_0 = 0$ one can obtain the rotating longitudinal flat M5-branes as found by Hyun and Shin [18]

$$\begin{aligned} Z_1(t) &= e^{-i\frac{\mu}{6}t}(x_4+ix_5)\,, \qquad Z_2(t)=e^{-i\frac{\mu}{6}t}(x_6+ix_7)\,,\\ [x_4,x_5]+[x_6,x_7] &= 0\,, \qquad [x_4,x_6]+[x_7,x_5]=0\,, \qquad [x_4,x_7]+[x_5,x_6]=0\,, \end{aligned} \tag{3.33}$$

where x_4, x_5, x_6, x_7 are time independent. The energy is given by $H = -\frac{\mu}{6}(M_{45}+M_{67})$, and hence, contrary to the conventional wisdom [21], the presence of the large longitudinal M5-branes do not always require the non-vanishing central charges when they rotate.

In any case, we do not know the most general infinite matrix solutions.

3.5 4/16 BPS configurations — type III

With the choice, $\lambda = (+++)$ and $(-+-)$, the corresponding 4/16 BPS equations are

$$\begin{aligned} \left[Z_0,X^3\right]+\frac{\mu}{3}Z_0 &= 0\,, \qquad \left[Z_2,X^3\right]-\frac{\mu}{6}Z_2=0\,, \qquad \left[Z_3,X^3\right]-\frac{\mu}{6}Z_3=0\,,\\ D_tZ_1+i\frac{\mu}{6}Z_1 &= 0\,, \qquad D_tX^i=0\,, \qquad D_tX^6=D_tX^7=D_tX^8=D_tX^9=0\,,\\ [Z_0,Z_2] &= 0\,, \qquad [Z_0,Z_3]=0\,, \qquad [Z_2,Z_3]=0\,,\\ \left[X^4,X^A\right] &= 0\,, \qquad \left[X^5,X^A\right]=0\,, \qquad [Z_0,\bar Z_0]+[Z_2,\bar Z_2]+[Z_3,\bar Z_3]-\frac{2}{3}\mu X^3=0\,. \end{aligned} \tag{3.34}$$

Note that the BPS equations themselves satisfy the Gauss constraint, $[X^4, X^5] = 0$, in this case too. The energy is saturated by the angular momenta and the central charges,

$$H = -\frac{\mu}{6}M_{45}+\frac{1}{2}(R_{1267}+R_{1289})\,, \tag{3.35}$$

with $M_{12} = M_{67} = M_{89} = R_{1245} = 0$.

Again we do not know the most general solutions. A particular solution we found involves a fuzzy sphere at the origin and a pair of hyperboloids. They are rotating on the $(4,5)$ plane. With the defining relation for a so(3) fuzzy sphere and a so(2,1) hyperboloid,

$$\begin{aligned} [J_1,J_2] &= iJ_3\,, \qquad [J_2,J_3]=iJ_1\,, \qquad [J_3,J_1]=iJ_2\,,\\ [K_1,K_2] &= -iK_3\,, \qquad [K_2,K_3]=iK_1\,, \qquad [K_3,K_1]=iK_2\,, \end{aligned} \tag{3.36}$$

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our solution reads

$$
\begin{aligned}
Z_1(t) &= e^{-i\frac{\mu}{6}t} z_1 1\,, \qquad A_0 = 0\,, \qquad X_1 = \frac{\mu}{3} P_0 J_1 P_0^\dagger\,, \qquad X_2 = \frac{\mu}{3} P_0 J_2 P_0^\dagger\,, \\
X_6 &= \frac{\sqrt{2}}{6}\mu P_1 K_2 P_1^\dagger\,, \qquad X_7 = \frac{\sqrt{2}}{6}\mu P_1 K_1 P_1^\dagger\,, \qquad X_8 = \frac{\sqrt{2}}{6}\mu P_2 K_2' P_2^\dagger\,, \\
X_9 &= \frac{\sqrt{2}}{6}\mu P_2 K_1' P_2^\dagger\,, \qquad X_3 = \frac{\mu}{3} P_0 J_3 P_0^\dagger + \frac{\mu}{6} P_1 K_3 P_1^\dagger + \frac{\mu}{6} P_2 K_3' P_2^\dagger\,,
\end{aligned} \tag{3.37}
$$

where P_0, P_1, P_2 are projection operators to the orthogonal spaces,

$$
P_0 = \sum_n |3n\rangle\langle n|\,, \qquad P_1 = \sum_n |3n+1\rangle\langle n|\,, \qquad P_0 = \sum_n |3n+2\rangle\langle n|\,, \tag{3.38}
$$

and K_1', K_2', K_3' form another so$(2,1)$ representation which can be different from the unprimed ones (3.36). We refer [22]–[25] for the details of the various so$(2,1)$ representations. The solution admits a Casimir operator,

$$
2X_1^2 + 2X_2^2 + 2X_3^2 - X_6^2 - X_7^2 - X_8^2 - X_9^2 = \text{constant} \times 1\,. \tag{3.39}
$$

3.6 8/16 BPS configurations — type I: su(4) singlet, various rotating objects

Three distinct sets of the 2/16 BPS equations are equivalent to the four distinct sets. Consequently there exits no genuine $\nu = 6/16$ classical BPS configuration. There are three inequivalent ways of summing 2/16 projection matrices, and hence three inequivalent sets of the 8/16 BPS equations.

The first type we consider deals with the choice, $\lambda = (+++), (++-), (+-+), (+--)$ so that quantum mechanically it corresponds to the su(4) singlet BPS multiplet [13]. The energy is saturated by a single angular momentum only,

$$
H = \frac{\mu}{3} M_{12}\,, \tag{3.40}
$$

and $M_{45} = M_{67} = M_{89} = R_{1245} = R_{1267} = R_{1289} = 0$.

The relevant 8/16 BPS equations are

$$
\begin{aligned}
D_t Z_0 + i[X^3, Z_0] - i\frac{\mu}{3} Z_0 &= 0\,, \qquad D_t X^3 = 0\,, \\
[Z_0, \bar{Z}_0] - \frac{2}{3}\mu X^3 &= 0\,, \qquad X^4 = X^5 = X^6 = X^7 = X^8 = X^9 = 0\,,
\end{aligned} \tag{3.41}
$$

while the Gauss constraint becomes

$$
[Z_0, [\bar{Z}_0, X^3]] + [\bar{Z}_0, [Z_0, X^3]] = \left(\frac{2}{3}\mu\right)^2 X^3\,. \tag{3.42}
$$

Some solutions of these BPS equations have been found by Bak [26] and also recently by Mikhailov [27] using different ansatz, but the general solutions are not known. The solutions include rotating D0-branes,

$$
X^3 = A_0 = 0\,, \qquad Z_0(t) = e^{i\frac{\mu}{3}t} \mathcal{Z}_0 : \text{diagonal matrix}\,, \tag{3.43}
$$

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rotating ellipsoidal branes with a real parameter, θ,

$$Z_0(t) = \frac{\sqrt{2}}{3}\mu e^{i\frac{\mu}{3}t}(\cos\theta\, J_1 + i\sin\theta\, J_2)\,, \qquad X^3 = A_0 = \frac{\mu}{3}\sin(2\theta)J_3\,, \tag{3.44}$$

rotating hyperboloids,

$$Z_0(t) = \frac{\sqrt{2}}{3}\mu e^{i\frac{\mu}{3}t}(\cosh\theta\, K_3 + i\sinh\theta\, K_1)\,, \qquad X^3 = A_0 = \frac{\mu}{3}\sinh(2\theta)K_2\,, \tag{3.45}$$

and rotating non-spherical giant gravitons like the fuzzy torus,

$$\begin{aligned}
&Z_0(t) \,=\, e^{i\frac{\mu}{3}t}\mathcal{Z}_0\,, \qquad X_3 = A_0 = \left(\frac{2}{3}\mu\right)^{-1}[\mathcal{Z}_0,\bar{\mathcal{Z}}_0]\,, \qquad [X^3,\mathcal{Z}_0] = \frac{\mu}{3}(\mathcal{Z}_0 - \theta(\bar{\mathcal{Z}}_0)^{-1})\,,\\
&\mathcal{Z}_0 \,=\, \sum_{n=1}^{N-1} q_n|n\rangle\langle n+1| + q_N|N\rangle\langle 1|\,,\\
&2|q_n|^2 - |q_{n+1}|^2 - |q_{n-1}|^2 = 2\left(\frac{\mu}{3}\right)^2\left(1-\theta|q_n|^{-2}\right)\,, \qquad q_0\equiv q_N\,, \qquad q_{N+1}\equiv q_1\,.
\end{aligned} \tag{3.46}$$

3.7 8/16 BPS configurations — type II: su(2) singlet, rotating fuzzy sphere

With the choice, $\lambda = (+++),(++-),(-++),(-+-)$, we deal with the su(2) singlet BPS multiplet [13]. The energy is saturated by a single angular momentum only,

$$H = -\frac{\mu}{6}M_{45}\,, \tag{3.47}$$

and $M_{12} = M_{67} = M_{89} = R_{1245} = R_{1267} = R_{1289} = 0$.

The corresponding BPS equations are

$$\begin{aligned}
&[X_i,X_j] - i\frac{\mu}{3}\epsilon_{ijk}X^k \,=\, 0\,, \qquad D_tX^i = 0\,, \qquad D_tZ_1 + i\frac{\mu}{6}Z_1 = 0\,,\\
&X^6 = X^7 = X^8 = X^9 \,=\, 0\,, \qquad [X^i,Z_1] = 0\,, \qquad [Z_1,\bar{Z}_1] = 0\,,
\end{aligned} \tag{3.48}$$

where the last equation agrees with the Gauss constraint in this case.

The solutions generically describe fuzzy spheres rotating on the $(4,5)$ plane,

$$X_i = \frac{\mu}{3}J_i\,, \qquad A_0 = 0\,, \qquad Z_1(t) = e^{-i\frac{\mu}{6}t}z_1 1\,. \tag{3.49}$$

Of course, when the representation of the fuzzy sphere is trivial, the solutions describe the D0-branes rotating on the $(4,5)$ plane [7].

3.8 8/16 BPS configurations — type III: static, large M2 or longitudinal M5

With the choice, $\lambda = (+++),(++-),(--+),(---)$, we obtain the "static" 8/16 BPS equations,

$$\begin{aligned}
D_tX^i = D_tZ_1 \,&=\, 0\,, \qquad X^6 = X^7 = X^8 = X^9 = 0\,,\\
[Z_0,X^3] + \frac{\mu}{3}Z_0 \,&=\, 0\,, \qquad [Z_1,X^3] - \frac{\mu}{6}Z_1 = 0\,,\\
[Z_0,Z_1] \,&=\, 0\,, \qquad [Z_0,\bar{Z}_0] + [Z_1,\bar{Z}_1] - \frac{2}{3}\mu X^3 = 0\,.
\end{aligned} \tag{3.50}$$

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The Gauss constraint is trivial surely. The energy is saturated by a central charge only,

$$H = \frac{1}{2} R_{1245} \,, \tag{3.51}$$

and $M_{12} = M_{45} = M_{67} = M_{89} = R_{1267} = R_{1289} = 0$. Thus, it describes static longitudinal large M5-branes stretching in the $1, 2, 4, 5$ directions or static large membranes stretching in the $3, 4, 5$ directions.

A class of solutions we found involves a fuzzy sphere and a hyperboloid. The hyperboloid is stretched in the $3, 4, 5$ directions. With the projection operators,

$$P_+ = \sum_n |2n\rangle\langle n| \,, \qquad P_- = \sum_n |2n+1\rangle\langle n| \,, \tag{3.52}$$

and the gauge choice, $A_0 = 0$, our solution reads

$$\begin{aligned} X_1 &= \frac{\mu}{3} P_+ J_1 P_+^\dagger \,, \qquad X_2 = \frac{\mu}{3} P_+ J_2 P_+^\dagger \,, \qquad X_4 = \frac{\sqrt{2}}{6} \mu P_- K_2 P_-^\dagger \,, \\ X_5 &= \frac{\sqrt{2}}{6} \mu P_- K_1 P_-^\dagger \,, \qquad X_3 = \frac{\mu}{3} P_+ J_3 P_+^\dagger + \frac{\mu}{6} P_- K_3 P_-^\dagger \,. \end{aligned} \tag{3.53}$$

Again we do not know the most general solutions.

3.9 16/16 BPS configurations: static fuzzy sphere

More than four sets of the 2/16 BPS equations have only the static fuzzy sphere as the common solution. Thus, there exits no genuine $\nu = 10/16, 12/16, 14/16$ classical BPS configuration. For the completeness we write the 16/16 BPS equations describing the static fuzzy spheres,

$$\begin{aligned} [X_i, X_j] - i\frac{\mu}{3}\epsilon_{ijk} X^k = 0 \,, \qquad D_t X^i = 0 \,, \\ X^4 = X^5 = X^6 = X^7 = X^8 = X^9 = 0 \,. \end{aligned} \tag{3.54}$$

4. Conclusion

We have obtained, in a systematic way, all the classical BPS equations which correspond to the quantum BPS states in the M-theory on a fully supersymmetric pp-wave.

The superalgebra of the M-theory matrix model shows that the BPS states always preserve pairs of supersymmetry, implying the possible fractions of the unbroken supersymmetry as $\nu = 2/16, 4/16, 6/16, \ldots$. Diagonalizing $\Gamma^{12}, \Gamma^{45}, \Gamma^{67}, \Gamma^{89}$ for the Cartan subalgebra, we were able to identify all the pairs of Killing spinors explicitly. There are eight of them and they are orthogonal and complete.

The key tool we employed was the projection matrix to the kernel space the Killing spinors form. The minimal, $\nu = 2/16$, projection matrices were constructed and written in terms of the anti-symmetric gamma matrix products. Three independent signs appearing in the expression make eight of them orthogonal and complete. The corresponding 2/16 BPS equations were then obtained from replacing the Killing spinor in the supersymmetry transformation formula by the projection operator. Expanding this formula by the gamma matrix products, we obtained eight sets of the 2/16 BPS equations of different sign choices. Up to the isometry group, SO(3) $\times$ SO(6), they are all equivalent. Similarly, the BPS equations of the higher fractions, $\nu = \mathcal{N}/16, \mathcal{N} = 4, 6, 8, \ldots$, can be obtained from the projection operator which is any $\mathcal{N}/2$ sum of the minimal ones. Effectively, the $\mathcal{N}/16$ BPS equations are equivalent to the $\mathcal{N}/2$ sets of the 2/16 BPS equations.

We found there are essentially one unique set of 2/16 BPS equations, three inequivalent types of 4/16 BPS equations, and three inequivalent types of 8/16 BPS equations, in addition to the 16/16 static fuzzy sphere. In particular, three of them correspond to the 4/16 su(2), 8/16 su(2) and 8/16 su(4) singlet BPS multiplets found in our previous work [13]. However, the 12/16 su(2) singlet BPS multiplets do not appear as classical configurations.

For each BPS configuration, we obtained the energy saturation formula in terms of the angular momenta and the central charges. The formula contains some useful informations such as the static properties, the rotational directions, and the stretched directions of the large objects, M2, M5. Our results show that all the classical su(2) singlet and su(4) singlet BPS configurations have vanishing central charges.

According to the superalgebra representation theory results [15], there can appear 2/16, 4/16, 6/16, 8/16, 12/16, 16/16 BPS states only in the supermultiplets. Our results show that at the classical level, 6/16 and 12/16 BPS configurations are missing.

In most of the cases we were not able to obtain the most general solutions, but we discussed at least one class of solutions in each case. For the su(2) singlet 4/16, 8/16 BPS equations we obtained the most general finite matrix solutions. They describe the rotating fuzzy spheres. Namely the fuzzy sphere is fully supersymmetric when it is static, half supersymmetric when it is rotating on a single plane, $(4,5)$, quarter supersymmetric when it is rotating on the three planes, $(4,5), (6,7)$ and $(8,9)$, demonstrating the supersymmetry breaking pattern as $16/16 \to 8/16 \to 4/16$. Some non-supersymmetric fuzzy sphere configurations have been studied in [28].

As for the D0-branes, when they rotate on the $(1,2)$ plane with the frequency, $\mu/3$, or $(4,5), (6,7)$, and $(8,9)$ planes with the frequency, $\mu/6$, we have the unbroken supersymmetry as

ν	rotating planes for D0			
$2/16$	$(1,2)$	$(4,5)$	$(6,7)$	$(8,9)$
$4/16_{\text{type I}}$	$(1,2)$	$(4,5)$		
$4/16_{\text{type II}}$		$(4,5)$	$(6,7)$	$(8,9)$
$8/16_{\text{type I}}$	$(1,2)$			
$8/16_{\text{type II}}$		$(4,5)$		

We found a class of solutions for the $4/16_{\text{typeIII}}$ BPS equations which consists of a fuzzy sphere and a pair of hyperboloids rotating on the $(4,5)$ planes. It would be interesting to find more mingled configurations which realize the curved longitudinal M5-branes.

The su(4) singlet 8/16 BPS equations have various known solutions in the literature as rotating D0-branes, ellipsoidal branes, hyperboloids and fuzzy torus. All of them rotate on the $(1,2)$ plane with the frequency, $\mu/3$, having the energy, $H = \frac{\mu}{3} M_{12}$.

The $8/16_{\mathrm{typeIII}}$ BPS equations are of unique interest since they are genuinely static equations. They describe static large longitudinal M5-branes stretching in the $1,2,4,5$ directions or static large membranes stretching in the $3,4,5$ directions. The energy is saturated by a single central charge, $H = \frac{1}{2} R_{1245}$.

Contrary to the conventional wisdom [21], the presence of the large longitudinal M5-branes do not always imply the non-vanishing central charges when they rotate.

In principle, one could obtain "$1/16, 3/16, 5/16, \ldots$ BPS equations" using the projection matrix method. The generic solution of these equations, if any, will correspond not to a single state in the supermultiplets, but to a linear combination of the states, for any choice of the Cartan subalgebra. Further investigation is required.

Our BPS equations are directly applicable to the BFSS matrix model or to the ten-dimensional super Yang-Mills theory. One simply needs to set $\mu = 0$ and replace $D_t X_A$, $-i[X_A, X_B]$ by F_{0A}, F_{AB}. We present the BPS equations in ten-dimensional super Yang-Mills theory in the appendix (A.12).

It would be interesting to see how the BPS configurations we obtained will appear in the IIA string theory on a pp-wave [29, 30] or in the DLCQ description of the longitudinal M5-branes on a pp-wave [31].

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A. BPS equations in 10D super Yang-Mills and other useful formulae

Here we show that *any solution of the BPS equations subject to the Gauss constraint satisfies the full equations of motion.*

First it is useful to note that under the supersymmetry transformations (2.16) the lagrangian transforms as

$$\begin{aligned}\delta\mathcal{L} &= \mathrm{tr}\left(\delta X^A \frac{\partial \mathcal{L}}{\partial X^A} + \delta \dot{X}^A \frac{\partial \mathcal{L}}{\partial \dot{X}^A} + \delta A_0 \frac{\partial \mathcal{L}}{\partial A_0} + \delta\psi_{\alpha\dot{\alpha}} \frac{\partial \mathcal{L}}{\partial \psi_{\alpha\dot{\alpha}}} + \delta\dot{\psi}_{\alpha\dot{\alpha}} \frac{\partial \mathcal{L}}{\partial \dot{\psi}_{\alpha\dot{\alpha}}} + \delta\bar{\psi}^{\alpha\dot{\alpha}} \frac{\partial \mathcal{L}}{\partial \bar{\psi}^{\alpha\dot{\alpha}}}\right) \\ &= \frac{d}{dt}\,\mathrm{tr}\left(\delta X^A \frac{\partial \mathcal{L}}{\partial \dot{X}^A} + i\delta\bar{\psi}^{\alpha\dot{\alpha}}\psi_{\alpha\dot{\alpha}}\right), \end{aligned} \tag{A.1}$$

from which we obtain the Noether charge of the supersymmetry (2.20).

For the BPS solutions satisfying $\delta\psi = \delta\bar{\psi} = 0$, the above relation reduces to

$$\mathrm{tr}\left[\Psi^\dagger \Gamma^A \mathcal{E}(t) \left(\frac{\partial \mathcal{L}}{\partial X^A} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{X}^A}\right)\right) + \Psi^\dagger \mathcal{E}(t) \frac{\partial \mathcal{L}}{\partial A_0}\right] = 0\,. \tag{A.2}$$

This equation holds for arbitrary Ψ, and hence

$$\Gamma^A \mathcal{E}(t) \left(\frac{\partial \mathcal{L}}{\partial X^A} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{X}^A} \right) \right)\Bigg|_{\Psi=0} + \mathcal{E}(t) \frac{\partial \mathcal{L}}{\partial A_0}\Bigg|_{\Psi=0} = 0\,. \tag{A.3}$$

At this point one can safely assume $\mathcal{E}(t)$ is bosonic. Then contracting with $\mathcal{E}(t)^\dagger \Gamma^B$ and using

$$\mathcal{E}(t)^\dagger \Gamma^B \Gamma^A \mathcal{E}(t) = \mathcal{E}(t)^T C^{-1} \Gamma^B \Gamma^A \mathcal{E}(t) = \mathcal{E}(t)^\dagger \Gamma^A \Gamma^B \mathcal{E}(t) = \delta^{AB} \mathcal{E}(t)^\dagger \mathcal{E}(t)\,, \tag{A.4}$$

we obtain

$$\mathcal{E}(t)^\dagger \mathcal{E}(t) \left(\frac{\partial \mathcal{L}}{\partial X^B} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{X}^B} \right) \right)\Bigg|_{\Psi=0} + \mathcal{E}(t)^\dagger \Gamma^B \mathcal{E}(t)\, \frac{\partial \mathcal{L}}{\partial A_0}\Bigg|_{\Psi=0} = 0\,. \tag{A.5}$$

This completes our proof.

The full equations of motion of the M-theory matrix model are

$$\begin{aligned} D_t D_t X_i + \left[X^A, [X_A, X_i]\right] + i\mu \epsilon_{ijk} X^j X^k + \left(\frac{\mu}{3}\right)^2 X_i - \left\{\bar{\psi}^{\alpha\dot{\alpha}}, (\sigma_i \psi)_{\alpha\dot{\alpha}}\right\} &= 0\,, \\ D_t D_t X_a + \left[X^A, [X_A, X_a]\right] + \left(\frac{\mu}{6}\right)^2 X_a - \frac{1}{2}\left\{\bar{\psi}^{\alpha\dot{\alpha}}, (\rho_a \tilde{\psi})_{\alpha\dot{\alpha}}\right\} - \frac{1}{2}\left\{\bar{\tilde{\psi}}^{\alpha}{}_{\dot{\alpha}}, (\bar{\rho}_a \psi)_{\alpha}{}^{\dot{\alpha}}\right\} &= 0\,, \\ i D_t \psi - \frac{\mu}{4}\psi - [X^i, \sigma_i \psi] - \left[X^a, \rho_a \tilde{\psi}\right] &= 0\,. \end{aligned} \tag{A.6}$$

Our choice of the euclidean six-dimensional gamma matrices (2.10) are off-block diagonal

$$\gamma^a = \begin{pmatrix} 0 & \rho^a \\ \bar{\rho}^a & 0 \end{pmatrix}, \qquad \rho^a \bar{\rho}^b + \rho^b \bar{\rho}^a = 2\delta^{ab}\,, \tag{A.7}$$

where the 4×4 matrices, $\rho^a, \bar{\rho}^b$ satisfy

$$\bar{\rho}^a = (\rho^a)^\dagger\,, \qquad (\rho^a)_{\dot{\alpha}\dot{\beta}} = -(\rho^a)_{\dot{\beta}\dot{\alpha}}\,, \qquad (\bar{\rho}^a)^{\dot{\alpha}\dot{\beta}} = -(\bar{\rho}^a)^{\dot{\beta}\dot{\alpha}}\,. \tag{A.8}$$

Using the U(4) symmetry, $\rho_a \to U \rho_a U^T, UU^\dagger = 1$, which preserves the anti-symmetric property of ρ_a, we can diagonalize $\gamma^{45}, \gamma^{67}, \gamma^{89}$ simultaneously

$$\begin{aligned} \rho^{45} &= i \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \rho^{67} = i \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \\ \rho^{89} &= i \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{A.9}$$

Explicitly we have (cf. [17])

$$\begin{aligned} \rho^4 &= \begin{pmatrix} i\epsilon & 0 \\ 0 & -i\epsilon^{-1} \end{pmatrix}, \qquad \rho^5 = \begin{pmatrix} \epsilon & 0 \\ 0 & \epsilon^{-1} \end{pmatrix}, \qquad \rho^6 = \begin{pmatrix} 0 & i\sigma^3 \\ -i(\sigma^3)^T & 0 \end{pmatrix}, \\ \rho^7 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \rho^8 = \begin{pmatrix} 0 & i\sigma^1 \\ -i(\sigma^1)^T & 0 \end{pmatrix}, \qquad \rho^9 = \begin{pmatrix} 0 & i\sigma^2 \\ -i(\sigma^2)^T & 0 \end{pmatrix}. \end{aligned} \tag{A.10}$$

Evaluating the anti-commutator of the supercharges to derive the supersymmetry algebra (2.26), one needs the following Fierz identities for the nine-dimensional gamma matrices, $(\Gamma^A)_{\bar\alpha}{}^{\bar\beta}, \bar\alpha, \bar\beta = 1, 2, \ldots, 16$,

$$\delta^{\bar\alpha}{}_{\bar\gamma}\delta^{\bar\beta}{}_{\bar\delta} - \delta^{\bar\alpha}{}_{\bar\delta}\delta^{\bar\beta}{}_{\bar\gamma} = \frac{1}{16}(C^{-1}\Gamma^{AB})^{\bar\alpha\bar\beta}(\Gamma_{AB}C)_{\bar\gamma\bar\delta} + \frac{1}{48}(C^{-1}\Gamma^{ABC})^{\bar\alpha\bar\beta}(\Gamma_{ABC}C)_{\bar\gamma\bar\delta}\,, \tag{A.11}$$
$$(\Gamma^{AB})_{\bar\alpha}{}^{\bar\gamma}(C^{-1}\Gamma_B)^{\bar\beta\bar\delta} + (C^{-1}\Gamma^{AB})^{\bar\beta\bar\delta}(\Gamma_B)_{\bar\alpha}{}^{\bar\gamma} + (\bar\gamma \leftrightarrow \bar\delta) = 2(\Gamma^A)_{\bar\alpha}{}^{\bar\beta}C^{-1\bar\gamma\bar\delta} - 2\delta_{\bar\alpha}{}^{\bar\beta}(C^{-1}\Gamma^A)^{\bar\gamma\bar\delta}\,.$$

The 2/16 BPS equations in ten-dimensional super Yang-Mills theory read with three independent signs, $\lambda_0^2 = \lambda_1^2 = \lambda_2^2 = 1$,

$$\begin{aligned}
F_{0A} + \lambda_0 F_{A3} &= 0\,, & \lambda_0 F_{12} + \lambda_1 F_{45} + \lambda_2 F_{67} + \lambda_1\lambda_2 F_{89} &= 0\,,\\
F_{14} + \lambda_0\lambda_1 F_{52} &= 0\,, & F_{15} + \lambda_0\lambda_1 F_{24} &= 0\,,\\
F_{16} + \lambda_0\lambda_2 F_{72} &= 0\,, & F_{17} + \lambda_0\lambda_2 F_{26} &= 0\,,\\
F_{18} + \lambda_0\lambda_1\lambda_2 F_{92} &= 0\,, & F_{19} + \lambda_0\lambda_1\lambda_2 F_{28} &= 0\,,\\
F_{46} + \lambda_1\lambda_2 F_{75} &= 0\,, & F_{47} + \lambda_1\lambda_2 F_{56} &= 0\,,\\
F_{48} + \lambda_2 F_{95} &= 0\,, & F_{49} + \lambda_2 F_{58} &= 0\,,\\
F_{68} + \lambda_1 F_{97} &= 0\,, & F_{69} + \lambda_1 F_{78} &= 0\,.
\end{aligned} \tag{A.12}$$

In addition there exists the Gauss constraint, $D^A F_{0A} = 0$. The generic BPS equations of the higher fractions, $\nu = 2/16, 4/16, 8/16, \ldots$, are ready to be obtained from this.

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